A Critical Assessment of the Resurrection of Logical Probability

J.J. Sylvia IV

The school of logical probability has generally been considered dead, however J. Franklin believes he has offered an analysis which will resurrect the position. This paper will argue against the assertions Franklin makes for that resurrection. Specifically two important counter-arguments are presented. In response to Franklin's assertion that some priors have no weight, but that others can be assigned weight based on a statistical syllogism and that this method is only available to the logical probabilist, it is argued that this formulation is based on the concept of frequency probability and thus does nothing to further the resurrection of logical probability. Second, in response to Franklin's assertion that background information for determining probability is ubiquitous and based on the conceptual framework, it is argued that this understanding would lead to a system where anything is either logically possible (1), or not (0), and that this is not useful for the decision making processes involved in science. Finally, I will suggest a possible direction for logical probability to take if the school is not to remain dead.

The basic idea underlying logical probability is that "probabilities can be determined a priori by an examination of the space of possibilities," (Hájek, 2007). The difficulty arises in determining these a priori possibilities. In other words, it is impossible to objectively assign a weight to many priors which would be used for determining logical probability. Franklin (p.279) gives a simple demonstration of this problem. If there is an unknown number of three types of balls mixed in a container (red, white, and blue), what is the initial probability that a person will draw out a white one? It could be interpreted as either 1/3 or 1/2.

One would reason it is 1/3 if they see it as drawing either a red, white, or blue ball; however, one could also view it as drawing either a white or a non-white ball. Therefore it seems that objectively determining the weight of a prior is impossible.

Franklin lays out three lines of defense for logical probability against this objection. His first defense is: “To maintain that any numbers given to the priors are, within limits, not to be taken seriously – because they are imprecise, fuzzy or vague, or unrobust in the face of evidence, or otherwise shaky for purely logical reasons,” (p. 280). What Franklin argues is that there are situations in which logical probability should not offer a number representing a prior weight. For example, with

> P (the moon is made of green cheese | Marilyn Monroe was murdered by the CIA)

Franklin argues that these two notions are not logically connected, and therefore no numerical probability should be assigned to them. Presumably he would like to apply this same analysis to the case of the balls in the urn. The probability is fuzzy or vague; therefore a logical probabilist need not assign any number whatsoever. Yet, the Marilyn Monroe example seems to suggest an easy number assignment: 0. If there is no logical connection between the two why would logical probability not easily be able to assign a 0 probability? Thus, instead of being a paradigmatic example of cases where objective priors are not necessary, this example only points out further the problems logical probability has in dealing with cases such as the balls in the container.

It seemed Franklin was on the right track in eliminating the need for qualitative measurements, but next Franklin takes an even more misguided step. He remains within the realm of probability and asserts that numbers are important for logical probability in many cases. He claims that on the other extreme, there are cases which provide a clear and precise number and that in these cases that number should be taken seriously. His example is:

P (Tex is rich | Tex is a Texan and 90% of Texans are rich) = 0.9

He states "It is to be noted that this assignment, natural as it is, is available to logical probabilists *only*," (p. 281, emphasis mine). This is a confused assertion. Instead of being an example of the best case of logical probability, this is an example of frequency probability. Hájek describes a simple version of frequency probability: "the probability of an attribute A in a finite reference class B is the relative frequency of actual occurrences of A within B." The probability of Tex being rich is determined through the use of frequency probability, not logical probability. Perhaps a friendly interpretation could assert that Franklin is trying to draw on aspects of frequency probability and incorporate them into logical probability, but this is ruled out when he says *only* the logical probabilist has this method at their disposal. This is simply not accurate.

Next Franklin uses anecdotal evidence from the *Talmud* in an attempt to show that logical probability is actually the best choice when there is little or no experience to draw upon. He gives the example:

> There follows a discussion of whether it is safe to marry a woman who has had two husbands die. It is felt that if the deaths were obviously due to some chance event, such as falling out of a palm tree, there is no need to

> worry, but if not, there may be some hidden cause in the woman, which the prospective husband would do well to take into account (p. 284).

Intuitively, this seems to be the correct analysis of the situation. Franklin asserts that it is only logical probability that can produce this intuitive answer. But how is this? If it is not known whether the husbands died by accident or by some wifely interference, then logical probability can offer up no suggestion that is not based on an arbitrary choice of the cause of the husbands' deaths. Frequency probability would also face a similar difficulty; however, if the cause is known then logical probability doesn't seem to be able to produce very certain numbers – or at least Franklin doesn't suggest how it could. On the other hand, frequency probability could offer a precise number. First, one would need to look at evidence of actual occurrences to derive the probability of a husband dying in a chance accident. For the sake of the example an arbitrary number can be picked: 1 out of 100. Then, the probability of this happening twice to the same woman would be the chance of it happening once, times the chance of it happening once again, or 1/100 * 1/100, which is 1 out of 10,000. To find the chance of it happening a third time, one would multiply 1/10,000 times 1/100, which would yield 1/1,000,000, or odds of one in a million. So, frequency probability can produce an answer that lines up with the intuitive example from the *Talmud* – if the accidents are chance occurrences then a third husband indeed has very little to worry about. It is not at all clear how logical probability would offer this same suggestion without relying on frequency calculations.

Of course, it may be that one could argue that the original intuition simply *is* the logical probability calculation. But why does one have this intuition? Most people know the odds of a freak accident such as a death or winning the lottery are fairly low for any individual person. Therefore the chances of it happening twice to the same person are even rarer. Even if the math that fully explains this is not understood, it is easy to see that twice achieving something that has 1 in a million odds is much less probable than achieving that same thing once. This "intuition" is really just elementary frequency probability calculations. One can still make judgments such as "more probable" or "less probable" using the concept behind frequency probability without having to know the exact numbers or how to process them, yet it seems that this is what Franklin would like to describe as logical probability. This is simply not accurate. All of the advantages he points to for logical probability are actually the advantages of frequency probability.

Franklin's second line of defense is:

> To maintain that although there is no uniquely specifiable prior, there is a principled distinction in the space of possible priors between reasonable and unreasonable ones, and that which is which is [sic] decidable, in large part at least, on logical grounds (p. 280).

This argument is aimed at defending the assignment of reasonable priors as objective. My objection is not that the assignment is not objective, but rather that it is based on frequency probability instead. Franklin says that none of the objections made of the assignment of relevant priors gives any reason to doubt the assignment of a uniform prior to the six possible outcomes of throwing a die (p. 286). But why is this selection made? In the "space of possible priors" all outcomes are possible. Perhaps there is something special about our universe relating to the number four, which will cause a die to always land with that side facing up. Why is this option unreasonable as a selection? Precisely because observation has shown that a die does not consistently land with the four facing up. This option for a logical probabilist is “reasonable” only because frequency probability has shown it to be so. So it may be granted that this selection is objective, but the more interesting and trying question is how does it differ from frequency probability? Unfortunately, this issue is not addressed.

Franklin’s final defense is:

> To maintain that in real cases, there is always a good deal of background information to be taken into account, so that reasonable debate about what the correct prior is may often be explained by differences in the background information being called into play (p. 280).

Franklin asserts that background information is ubiquitous. First, symmetry arguments may be weak or strong. For example (p. 287), a die that looks symmetrical by glance only is a weak symmetry argument for equal probability, whereas a die that has been measured and weighed and thus shown more diligently to be symmetrical would provide a strong symmetry argument for equal prior probability. Hiding behind this example is a simple epistemological issue. A logical probabilist trying to determine the probability of a die would need to know, just as much as a frequency probabilist, whether the die was actually symmetrical or not before he is able to pronounce on any probabilities whatsoever. Franklin takes this epistemological position, labels it the ubiquity of background information for priors, and somehow takes this to be a defense for logical probability. A lack of knowledge about a situation when trying to determine probability is a problem for any theory of probability, not a defense of it.

Perhaps the point is better seen through the second example:

> Consider, however, the generalisation, “All humans are less than 5 metres tall”. This is confirmed by present observations of people, all of whom have been observed to be less than 5 metres tall. Suppose then an expedition returns from a previously unexplored jungle and reports, with suitable evidence, that they have observed a human 4.99 metres tall. On this evidence, the probability that all people are less than 5 metres tall is nearly zero, although the generalisation still has only positive instances (p.287) [sic].

Finally one is able to see here an example where logical probability would certainly differ from frequency probability. A frequency probabilist would count the 4.99 meters tall human as simply another instance of a human less than 5 meters tall, and this would be more evidence that “all humans are less than 5 meters tall.” The logical probabilist takes another view. Franklin wants to use this as an example of the ubiquity of background information: there is information hidden in the structure of the concept of length. “Length, as everyone who uses the concept knows, is something that admits of continuous variation, which means that the existence of something 4.99 meters tall makes it probable that there is a similar thing at least 5 meters tall,” (p. 287). But does it? This is an important open question.

Assuming the human species is finite and will not continue forever, there would necessarily be a tallest person who ever existed. It is possible that this tallest person happened to be 4.99 meters tall. This person could represent one freak mutation or accident that is far outside of the normal range for humans. If there is not a trend showing his local population or family line to be growing consistently taller across time, then it does not seem necessarily *more probable* that there be a human 5 meters tall. Is this the ubiquity of background information to which Franklin refers? Depending on how one analyzes the situation, the probability can seem very different, even within logical probability itself. So where is the utility in this? Or perhaps a more difficult question, where does the logic of logical probability enter into probability determinations? It seems not to offer any advice about *which* background information to take as important, but merely asserts that there are different points of view. At least with mathematic probability or frequency probability one knows beforehand why the specific probability is given. Rather than give any precise probabilities, logical probability seems better suited at telling one whether something is logically possible (1) or not (0). But this has very limited use, especially in regards to scientific decision making. It also seems less than parsimonious to develop a system that can only tell one if something is logically possible or not when this can be done just as well without that system.

Why does this confusion arise in the first place? Franklin wants to show that just about every position on probability is actually secretly espousing the logical probabilist perspective. This would include frequency probabilists. In his explanation of why frequency probability is actually secretly logical probability, it begins to become clear where the real problem arises. Franklin argues:

> Von Mises and Reichenback regarded probability as only properly definable in a infinite sequence of throws, or a “collective”, or as a limit of relative frequencies in a finite class. As was always recognized, this created a difficulty

> in explaining how probability so defined could be relevant to belief, decision, or action in the single case. (Franklin, p. 299).

Reichenbach proposes an answer to this problem, but Franklin takes this answer to contain an overt proportional syllogism and thus default to logical probability. However, this “difficulty” is not a difficulty with frequency probability, per se, but is rather a difficulty of epistemic warrant for beliefs involving *any* probability. Through the arguments presented here by Franklin, it becomes clear that perhaps the notion he is arguing for is not one of a system of probability at all, but rather a system of epistemic warrant. If this were the case, it would make much more sense that many of his examples are based on or equal to frequency probability.

This confusion, now clearly seen, becomes evident in other passages as well. To begin with, Ramsey argues against Keynes formulation of logical probability:

> There really does not seem to be any such things as the probability relations he [Keynes] describes. He supposes that, at any rate in certain cases, they can be perceived; but speaking for myself I feel confident that this is not true. I do not perceive them, and if I am to be persuaded that they exist it must be by argument; moreover I shrewdly suspect that others do not perceive them either, because they are able to come to so very little agreement as to which of them relates any two given propositions. (Ramsey, 1926, p.57, quoted by Franklin, p. 289).

Franklin responds to this criticism: “Juries are able to agree on verdicts, at least in clear-cut cases, which would happen extremely rarely if there were indeed the general disagreement Ramsey asserts,” (p. 289). Again, this seems to be confusion of talk about epistemology for talk about probability. Agreement in belief does not equate ontological reality. For example, there have been plenty of cases where juries have agreed on verdicts and convicted a man who was later proven innocent by forensics evidence. Ramsey seems to be correct in asserting that there exists very little agreement, but even if the agreement were to exist, this would be an epistemological claim and not a metaphysical claim.

After an examination of Franklin's arguments for logical probability, it appears that the system he hopes to save offers up nothing of value to the field of probability itself because it always forced to draw upon the methods of some other system of probability. I have suggested frequency probability has the ability to handle all of the work the proposed version of logical probability would do, but I do not want to limit myself to only this framework. Patrick Maher attempts to lay out the concept of inductive probability, but in doing so, his clarifications are able to further the analysis showing that logical probability is not a system of probability at all.

Maher sets out to show why various systems typically identified with probability are not the same as the idea of inductive probability he would like to advance. In discussing logical probability he notes that "it is often taken to mean 'uniquely rational degree of belief'", (Maher, p. 193). The typical view for this position is:

> (1) The probability of H given E is the degree of belief in H that is rational for a person whose total evidence is E.

A counter-example is quickly offered:

> Suppose that X is a competitor in a sports event and knows

> that he will perform better if he has a high degree of belief that he will win. Then it may be rational for X to have a high degree of belief that he will win, even if the inductive probability of this, given X's evidence, is low. (Maher, p. 188).

The suggested fix for the rational degree of belief view would then be:

> (2) The probability of H given E is the degree of belief in H that is *epistemically* rational for a person whose total evidence is E.

Again, what we have is logical probability being shown to be a theory of epistemic warrant, and not of probability. This being the case, calling the system logical probability is a misnomer.

The problem, then, is that Franklin attempts to take the notion of logical probability much too far. Throughout his paper he argues that subjective Bayesians, frequentists, and Karl Popper are all secret logical probabilists. As stated it would seem that holding any one of these three positions should be mutually exclusive from holding the position of a logical probabilist. Franklin believes this means that all these systems are wrong, while logical probability is the proper system. However, if these three systems are matters of probability, while (the unfortunately named) logical probability is really a system of epistemic warrant, then perhaps the positions are not mutually exclusive after all. Perhaps one could, after all, be both a frequency probabilist *and* a logical probabilist consistently. This does not seem possible through the work of Franklin, but if the emphasis of logical probability is shifted back to its epistemic warrant, then perhaps this compatibility could be demonstrated. Instead of asking which prior is more logically probable, it may be more proper to ask which prior has more epistemic warrant.

This should be seen as an important distinction which leaves a lot of questions to be answered. Vincenzo Fano takes a beginning step toward addressing how this more accurate version of logical probability would function. “We would all agree that Newton’s gravitation law is more confirmed than the astral influences on our character and behaviors. In spite of this it is not easy to *quantify* such a difference as a degree of confirmation,” (Fano, p.1). Would frequency probability alone offer strong enough epistemic warrant for a particular belief? This does not seem likely, at least not in every case, although there are clear cut cases where frequency probability can offer strong epistemic warrant, including most of the cases Franklin mentions.

Outside of these easy cases there are much more difficult questions to answer using probability. How would one attempt to quantify the astral influences on character and behavior? Perhaps horoscopes could be examined. We could attempt to discover for how many people a horoscope has been accurate in the past. Yet even in the framing of this question the answer is already biased. Perhaps by mere chance, ninety percent of people have had a single horoscope be accurate once in their lives. According to our current question, it might seem as if horoscopes are pretty accurate. But this question does not take into account the hundreds or perhaps thousands of days that horoscopes are *not* accurate for each of these people, nor does it speak to the notion that horoscopes are so general that they could possibly be interpreted to apply to everyone all time. There are other difficulties with trying to gather frequency statistics as well. Problems such as sample size and biased questions show that there may even be more than one frequency to take into account. These are the very situations where a system determining epistemic warrant would be useful. Frequency probability does not answer every question posed, nor does logical probability better address questions of probability, as Franklin asserts. Instead, the two are very different concepts which should be kept distinct. Let each answer the questions it can best answer.

What direction then does that leave for logical probability to take? I have tried to show where it definitely *should not* go. It should not aim to replace or overtake systems such as frequency probability which already perform their specific tasks well. Fano suggests that perhaps logical probability works best when comparing one or more alternatives for rational degrees of belief, rather than trying to place a numerical value on every possibility, as frequency probability already does. However this leads to its own problems:

> Though it is more epistemologically reasonable to deal with most probabilities in comparative terms, we have not yet been able to define a qualitative updating of probability. On the contrary, an updating of probability is possible only if the latter is quantitative. Nonetheless it is reasonable to maintain that only cases very simple from the cognitive point of view allow a reasonable application of a probability measure. (Fano, p. 6)

Perhaps the trouble comes in even holding on to a notion of quantitative measurement for evaluating degrees of rational belief. Let us take an historical example. French poet Lamartine said that one of his famous poems came to him fully in a single flash of inspiration. Someone living during the same time period as Lamartine is trying to determine if the poem was a flash of inspiration or was worked out meticulously. The only obvious evidence at the time is the word of

Lamartine, and there is no evidence that his word should not be believed. Therefore it seems to be a rational belief that the poem was conceived in a flash. However, upon Lamartine’s death, many drafts and versions of the poem in question are discovered, and the evidence now points to the notion that this poem was very carefully worked out. It seems obvious with this updated evidence that it is rational to believe the poem was worked out rather than conceived in a flash. Is it necessary to quantify this claim, or is there rather some statement of quality being made? This qualitative claim for epistemic warrant, rather than the direction of quantitative probabilities in which Franklin tries to steer the system, seems to be the best direction for logical probability to pursue if it is not to remain a dead movement.

References

Fano, Vincenzo (2004). A Critical Appraisal of Comparative Probability. http://philsci-archive.pitt.edu/archive/0003019/01/scazzieriingl.doc. Accessed December 3, 2007.

Franklin, J. (2001). Resurrecting Logical Probability. *Erkenntnis* 55: 277-305.

Hájek, Alan (2007). Interpretations of Probability. http://plato.stanford.edu/entries/probability-interpret/. Accessed 11/30/07.

Maher, Patrick (2005). The Concept of Inductive Probability. *Erkenntnis* 65: 185-206.